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Comparative study of various computational models for triple moments of velocity and scalar in second-order closure

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Abstract—A comparative assessment of several computational turbulence models for third-order diffusive transport terms, $\overline{u_i u_j \theta}$ and $\overline{u_i \theta^2}$, in second-order closure equations has been carried out by applying the models to various non-isothermal turbulent flows. The second-order quantities appearing in the models are adopted from directly measured values. The models tested in the present study are: conventional simple gradient model, eddy-damped quasi-normal approximation model, and Weinstock's theoretical model which is derived by formally integrating the Navier–Stokes equation [J. Weinstock, *J. Fluid Mech.* **202,** 319–338 (1989)]. It is rather a surprise to find that the simple gradient model performs equally or even better than the other more complicated ones for the scalar variance diffusion, $\overline{u_i \theta^2}$. However, it is found that the computational model for the scalar flux diffusion, $\overline{u_i u_j \theta}$, must include the shear-gradient contribution in addition to the simple gradient model. Moreover, a buoyancy correction method is proposed to take into account the buoyancy effect in the gradient-type models.

1. INTRODUCTION

THE THIRD-ORDER diffusive transport terms, $\overline{u_i u_j \theta}$ and $\overline{u_i \theta^2}$, which appear in the second-order closure equations for the scalar flux ($\overline{u_j \theta}$) and the scalar variance ($\overline{\theta^2}$), respectively, play significant roles in spacially transporting the second-order quantities of interests in, for example, a thermally stratified atmospheric boundary layer [1], the thermal dispersion field behind an elevated line heat source [2, 3] and a turbulent thermal convection [4].

During its infant period of the second-order modelling, such higher-order moments were approximated by simple gradient-type models [5–9]. However, since it was decidedly shown by Zeman and Lumley [10] that the evolution process of a buoyancy-driven mixed layer in the upper atmosphere depends crucially on the third-order transport, serious attempts have been made at deriving elaborate transport models by Dekeyser and Launder [11], Lumley [12] and Weinstock [13], which are in order of increasing complexity. One of the important questions is whether such elaboration of the model formulation results in a better prediction accuracy or not.

The present study is addressed to finding an objective answer to the above question by testing the models against a number of available experiments, and to proposing a desirable method to modify the previous simple gradient-type model for flows under a non-negligible buoyancy effect.

2. THIRD-ORDER DIFFUSIVE TRANSPORT MODELS

2.1. *Simple gradient-type model*

The conventional simple gradient-type model for the scalar flux diffusion term, $\overline{u_i u_j \theta}$, takes the following form (e.g. Launder [9]):

$$-\overline{u_i u_j \theta} = C_s \tau_v \left(\overline{u_j u_k} \frac{\partial \overline{u_i \theta}}{\partial x_k} + \overline{u_i u_k} \frac{\partial \overline{u_j \theta}}{\partial x_k} \right). \quad (1)$$

The above model may be derived from the exact equation for $\overline{u_i u_j \theta}$ by neglecting convective transport, molecular dissipation, generation due to mean field gradients and the contribution of pressure transport. Further, it is implicitly assumed that the generation due to the shear stress gradient is also negligible compared with that due to the scalar flux gradient.

As for the third-order diffusive scalar transport term, $\overline{u_i \theta^2}$, the following model has been widely adopted in calculating atmospheric boundary layer flows (e.g. Wyngaard [14]):

$$-\overline{u_i \theta^2} = C_s \tau_v \overline{u_i u_k} \frac{\partial \overline{\theta^2}}{\partial x_k} \quad (2)$$

which can be obtained in the manner analogous to the above description for the model (1).

The model constant C_s has been assigned to a number ranging from 0.11 to 1.0, i.e. 0.11 [2], 0.14 [15], 0.15 [13], 0.2 [9], 0.3 [8], and 1.0 [12]. Excluding the

NOMENCLATURE

- $[A]$ coefficient matrix
- a, b, C_1, C_s, C_θ model constants
- g_i gravitational acceleration in the direction i
- h source height
- K_{ij} turbulent eddy coefficient tensor
- L_d half-width of mean temperature profile
- N Brunt–Väisälä frequency, $(g\beta\,\mathrm{d}T/\mathrm{d}y)^{1/2}$
- q^2 twice the turbulent kinetic energy, $\overline{u_i u_i}$
- T mean temperature
- U mean velocity
- u velocity scale, $u^2 = \frac{1}{3}\overline{u_i u_i}$
- u_* friction velocity
- u, v, w velocity fluctuations in axial, vertical, and lateral directions, respectively
- x, y coordinate axes in axial and vertical directions, respectively.

Greek symbols
- β thermal coefficient of cubical expansion
- ε dissipation rate of turbulent kinetic energy
- ε_θ dissipation rate of temperature variance
- θ temperature fluctuation
- θ_* temperature scale
- ν_t turbulent eddy diffusivity
- τ_b buoyancy time scale
- τ_0 time scale for Weinstock's model in equation (7)
- τ_v dynamic time scale
- τ_θ thermal time scale.

Subscripts
- eff effective
- p peak.

Superscripts
- $\overline{(\;)}$ conventional time average
- $'$ r.m.s. value.

highest value which has not been used by others, C_s is made equal to the average value of 0.18 in the present calculation.

2.2. *Dekeyser and Launder's model*

Dekeyser and Launder [11] have proposed several alternative models for $\overline{u_i u_j \theta}$ and $\overline{u_i \theta^2}$, which are derived from the exact transport equations for $\overline{u_i u_j \theta}$ and $\overline{u_i \theta^2}$, respectively, using a number of simplifying assumptions. The differences between the alternatives arise from the inclusion or exclusion of a dissipative term and a pressure-isotropization term in the modelling process. However, since they were not able to conclude that any of the models performed distinctively better than the others, the following simplest models are tested in the present comparative study:

$$-\overline{u_i u_j \theta} = C_\theta \tau_v \left(\overline{u_j u_k}\frac{\partial \overline{u_i \theta}}{\partial x_k} + \overline{u_i u_k}\frac{\partial \overline{u_j \theta}}{\partial x_k} + \overline{u_k \theta}\frac{\partial \overline{u_i u_j}}{\partial x_k} \right) \quad (3)$$

$$-\overline{u_i \theta^2} = C_\theta \tau_v \left(\overline{u_i u_k}\frac{\partial \overline{\theta^2}}{\partial x_k} + 2\overline{u_i \theta}\frac{\partial \overline{u_k \theta}}{\partial x_k} \right) \quad (4)$$

where $C_\theta = 0.11$ and $\tau_v \equiv q^2/2\varepsilon$ is the turbulent velocity time scale.

It may be noted by comparing these models with those of (1) and (2) that the last terms in (3) and (4) are added to the simple gradient-type models. The appearance of these terms implies that the contribution of the shear stress gradient to the generation of $\overline{u_i u_j \theta}$ and that of the scalar flux gradient to the generation of $\overline{u_i \theta^2}$ are retained in formulating the models.

2.3. *Lumley's model*

Neglecting substantial derivatives, production and rapid terms, and higher-order cumulants in the transport equations for $\overline{u_i u_j \theta}$ and $\overline{u_i \theta^2}$, Lumley [12] has presented algebraic equations for the third-order terms in tensorial form. Explicit expressions for $\overline{u_i u_j \theta}$ and $\overline{u_i \theta^2}$ from their equations are as follows:

$$-\overline{u_i u_j \theta} = \frac{1}{0.5C_1 + C_\theta}\tau_v A_{ij} - \frac{2 - C_1}{3(C_1 + 2C_\theta)(1 + C_\theta)} \times \tau_v A_{kk}\delta_{ij} \quad (5)$$

$$-\overline{u_i \theta^2} = \frac{1}{1 + C_\theta \tau_\theta/\tau_v}\tau_\theta \left(\overline{u_i u_k}\frac{\partial \overline{\theta^2}}{\partial x_k} + 2\overline{u_i \theta}\frac{\partial \overline{u_k \theta}}{\partial x_k} \right) \quad (6)$$

where $C_1 = 3.5$, $C_\theta = 7.5$,

$$A_{ij} = \overline{u_j u_k}\frac{\partial \overline{u_i \theta}}{\partial x_k} + \overline{u_i u_k}\frac{\partial \overline{u_j \theta}}{\partial x_k} + \overline{u_k \theta}\frac{\partial \overline{u_i u_j}}{\partial x_k}$$

and $\tau_\theta \equiv \overline{\theta^2}/2\varepsilon_\theta$, the turbulent thermal time scale. Since the inhomogeneity in temperature is assumed to be weak, the buoyancy effect is neglected in the models.

2.4. *Weinstock's model*

The above models were derived based on the eddy-damped quasi-normal approximation as applied to single-point moments. But, it was found by Wyngaard [14] that there is a significant discrepancy for eddy-damping in inhomogeneous flows. In order to avoid such basic assumptions inherent in the eddy-damping method, Weinstock [13] performed a formal integration of the Navier–Stokes equation that permits one to express single-time third-order moments in terms of two-time fourth-order moments, which are then evaluated by neglect of two-time fourth-order cumulants. Simplifying assumptions he made are: (a) all quantities are assumed to vary slowly in space and time compared with the integral length and the Lagrangian time scale; (b) the anisotropy is small; (c) average quantities may vary in the vertical direction (horizontal stratification) only; and (d) the mean flow is unidirectional in the form, $U = [U(x_3), 0, 0]$.

The final models for the third-order transport terms are represented in matrix form:

$$[\overline{u_2^3}, \overline{u_2 u_1^2}, \overline{u_2^2\theta}, \overline{u_3\theta^2}]^T = -\tau_0[A] \times \left[\frac{\partial \overline{u_2^2}}{\partial x_2}, \frac{\partial \overline{u_1^2}}{\partial x_2}, \frac{\partial \overline{u_1 u_2}}{\partial x_2}, \frac{\partial \overline{u_2\theta}}{\partial x_2}, \frac{\partial \overline{u_1\theta}}{\partial x_2}, \frac{\partial \overline{\theta^2}}{\partial x_2}\right]^T. \quad (7)$$

Here, τ_0 is a time scale defined by

$$\tau_0 = \frac{0.15\tau_v}{1+0.17N^2\tau_v^2 H(N^2)} \quad (8)$$

for high Reynolds number, $N^2 \equiv (g\beta\, \partial T/\partial x_2)$ is the Brunt–Väisälä frequency, $H(N^2)$ is the Heaviside step function and β is the thermal expansion coefficient. The coefficient matrix $[A]$ has 4×6 elements which are functions of N^2, T and second-order terms (for details, see ref. [13]).

2.5. *Modification of simple gradient-type models to include the buoyancy effect*

In the k–ε turbulence model, the eddy coefficients of the third-order diffusive transport terms are represented by the turbulent eddy diffusivity ν_t/σ_t where σ_t is the turbulent Prandtl number. Such practice is based on the assumption that the second-order turbulence quantities are transported in a manner similar to the momentum and heat.

For horizontal flows under an appreciable buoyancy effect, the algebraic stress model of Ljuboja and Rodi [16] or Chung and Sung [17] can be manipulated to obtain the eddy diffusivity, ν_t, in the following form:

$$\nu_t = C_s \frac{\tau_v}{1+aN^2\tau_v^2}(\overline{v^2} + b\tau_\theta \beta g \overline{v\theta}) \quad (9)$$

where v is in the direction opposite to the gravitational vector. Model constants a, b and C_s are functions of other constants. Adopting the constants in Ljuboja and Rodi [16], $a \approx 0.075$, $b \approx 0.17$ and $C_s \approx 0.23$. The expression (9) together with the time scale in equation (8) strongly implies that a certain turbulent time scale in a buoyancy-affected flow field must depend on the time scale ratio $N\tau_v$ and that the velocity scale squared for turbulent transport processes be a function of buoyancy. For a thermally stable layer $N^2 > 0$ and $\overline{v\theta} < 0$, hence the turbulent eddy coefficient (9) becomes smaller than that in neutral stratification and for a thermally unstable one, vice versa; these characteristics are physically realistic.

In order to generalize the above observation to the turbulent eddy coefficient K_{ij} of the third-order diffusive transport terms, the buoyancy relaxation time scale, $\tau_b \equiv 3\varepsilon_\theta/(N^2\overline{\theta^2})$, and the effective buoyant Reynolds stress,

$$\overline{u_i u_j}_{\text{eff}} = \overline{u_i u_j} - b\beta(g_i\overline{u_j\theta} + g_j\overline{u_i\theta})\tau_\theta,$$

both of which are proposed by Lumley *et al.* [1, 18], are introduced to construct the turbulent eddy coefficient in a form

$$K_{ij} = C_s \frac{\tau_v}{1+aH(N^2)\tau_v/\tau_b}[\overline{u_i u_j} - b\beta(g_i\overline{u_j\theta} + g_j\overline{u_i\theta})\tau_\theta], \quad (10)$$

where, g_i is the gravitational vector field, $g_i = (0, -g, 0)$. The Heaviside function $H(N^2)$ is also introduced in the above formulation as in model (8) due to the reason discussed in Weinstock [13]. From the physical point of view, the eddy coefficient should be positive. Therefore, when the bracketed term in equation (10) becomes negative due to strong downward heat flux very near a heat source or a sink, for example, K_{ij} must be made equal to zero.

Accordingly, the simple gradient-type models may be modified to include the buoyancy effect by utilizing the above buoyancy-affected eddy coefficient as follows:

$$-\overline{u_i u_j \theta} = C_s \frac{\tau_v}{1+aH(N^2)\tau_v/\tau_b}\left(\overline{u_i u_k}_{\text{eff}}\frac{\partial \overline{u_j\theta}}{\partial x_k} + \overline{u_j u_k}_{\text{eff}}\frac{\partial \overline{u_i\theta}}{\partial x_k}\right) \quad (11)$$

$$-\overline{u_i\theta^2} = C_s \frac{\tau_v}{1+aH(N^2)\tau_v/\tau_b}\overline{u_i u_k}_{\text{eff}}\frac{\partial \overline{\theta^2}}{\partial x_k}. \quad (12)$$

Since there are not enough data to finely adjust the model constants a and b, $a = 0.12$ has been selected simply by averaging the values in the models (8) and (9). It is worth noting that this value coincides with the corresponding one in the streamline curvature model of Chung *et al.* [19]. Model constant b has been recommended by Zeman and Lumley [10] to be about 3.0 which is adopted in the present work. And, for consistency, the value of 0.18 is also used for C_s as that in the simple gradient-type models (1) and (2).

Now a discussion about the time scale in equation (10) may be in order. Zeman and Lumley [10] used a composite time scale

$$\tau_c \equiv \left(\frac{1}{\tau_v} + \frac{0.28}{\tau_\theta}\right)^{-1}$$

in place of τ_v in the numerator of equation (10). In most non-isothermal flows, however, the time scale ratio τ_θ/τ_v lies in a range 0.35–0.6 with an average value of about 0.5 [20]. Therefore, there is not much difference between τ_v and τ_c in numerically calculating the eddy diffusivities, and for simplicity τ_v is preferred in the present study.

3. RESULTS AND DISCUSSION

A total of five models for third-order scalar flux and scalar variance diffusive transports have been presented in the previous section in terms of second-moment quantities, their derivatives and the mean temperature. In order to test these models against measurements, the necessary data must all be available in the experiments. In the present comparisons, the following recent experiments are selected due to their relative completeness of the information about turbulence quantities which have been published through the literature: Raupach and Legg [2] measured the turbulent scalar dispersion field from an elevated line heat source in a turbulent boundary layer. Karnik and Tavoularis [21] investigated experimentally the heat diffusion from a continuous line source placed in a uniformly sheared homogeneous turbulent flow. At nearly the same time, a similar experiment had been performed by Chung and Kyong [3] in a weakly sheared homogeneous turbulent flow. And, quiet recently, Veeravalli and Warhaft [22] obtained detailed turbulence data in the thermal dispersion field from a heated line source placed in the central region of a turbulent mixing layer having no mean shear. The measured distributions of the triple moments are compared with predicted results by all models in Section 2. In doing so, the second-order quantities and the mean temperature data appearing in the various model equations are directly taken from the experimental values.

Comparisons between the experiments and the model predictions are provided in Figs. 1, 2 and 3 for $\overline{v\theta^2}$, $\overline{v^2\theta}$ and $\overline{uv\theta}$, respectively. The scalar variance diffusions shown in Figs. 1(b) and (c) reveal that the appearance of the second term in equations (4) and (5) does not improve the predictions. Rather, it severely deteriorates them near the zero-crossing in the central

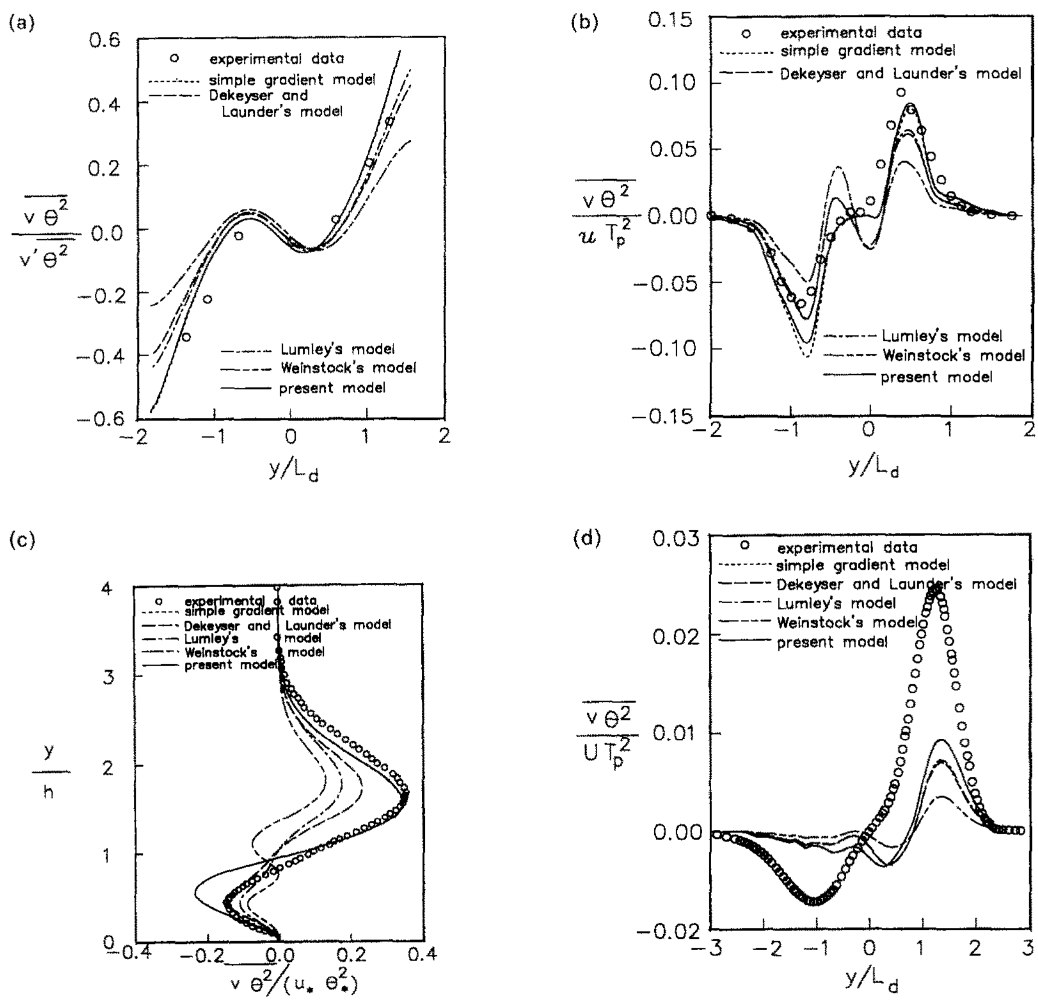

FIG. 1. (a) Model predictions and experimental (Karnik and Tavoularis' [21]) data for the vertical transport of temperature variance, $\overline{v\theta^2}$. (b) Model predictions and experimental (Chung and Kyong's [3]) data for the vertical transport of temperature variance, $\overline{v\theta^2}$. (c) Model predictions and experimental (Raupach and Legg's [2]) data for the vertical transport of temperature variance, $\overline{v\theta^2}$. (d) Model predictions and experimental (Veeravalli and Warhaft's [22]) data for the vertical transport of temperature variance, $\overline{v\theta^2}$.

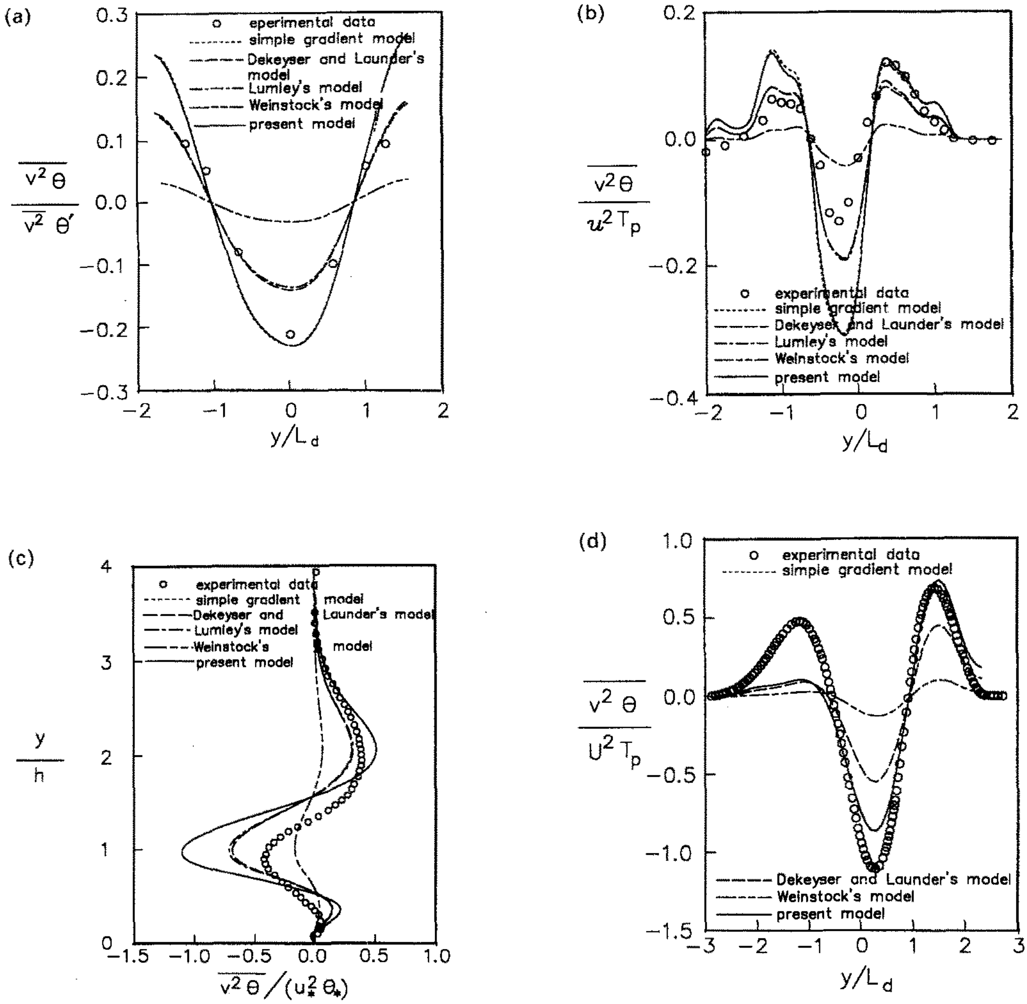

FIG. 2. (a) Model predictions and experimental (Karnik and Tavoularis' [21]) data for the vertical transport of the vertical heat flux, $\overline{v^2\theta}$. (b) Model predictions and experimental (Chung and Kyong's [3]) data for the vertical transport of the vertical heat flux, $\overline{v^2\theta}$. (c) Model predictions and experimental (Raupach and Legg's [2]) data for the vertical transport of the vertical heat flux, $\overline{v^2\theta}$. (d) Model predictions and experimental (Veeravalli and Warhaft's [22]) data for the vertical transport of the vertical heat flux, $\overline{v^2\theta}$. The vertical scale has been multiplied by 1000.

region. In the non-negligible buoyant flow in Fig. 1(b), it can be demonstrated clearly that the time scale modification by buoyancy improves the predictions both in the stable region ($y < 0$) and the unstable one ($y > 0$). For Karnik and Tavoularis' data in Fig. 1(a), the models of Weinstock, and Dekeyser and Launder yield the least satisfactory results. In the highly inhomogeneous flow of Veeravalli and Warhaft, all models fail to represent the measured data. Nevertheless, it can be said that the simple gradient-type models yield better profiles in reasonable agreement with the experiments.

Secondly, the predicted scalar flux diffusions in the vertical direction, $\overline{v^2\theta}$, are compared with the measured profiles in Figs. 2(a)–(d). Experimental data of Karnik and Tavoularis in Fig. 2(a) and Veeravalli and Warhaft in Fig. 2(d) represent the most favourable comparisons with predictions by the simple gradient-type models. Although, the computed values in the central regions by the simple gradient-type models in Fig. 2(b) are too low, the time scale modification to the model notably shifts the profile in the correct direction in both the stable and unstable regions. Significant shear is present in the central regions of the experiments of Figs. 2(b) and (c). Therefore, it may be concluded from the comparisons in Figs. 2(b) and (c) that the addition of the shear-gradient term to the simple gradient model, as in models (3) and (5), improves the estimations of the scalar flux diffusion in the cross-stream direction.

Finally, Figs. 3(a)–(c) illustrate the computational performances of the various models for the streamwise diffusion of vertical heat flux, $\overline{uv\theta}$. Owing to the lack of necessary data for computing the model, that of Weinstock is not included in the comparison. As for Karnik and Tavoularis' data in Fig. 3(a), both the simple gradient-type model and the modified one perform better. However, for flows with an appreci-

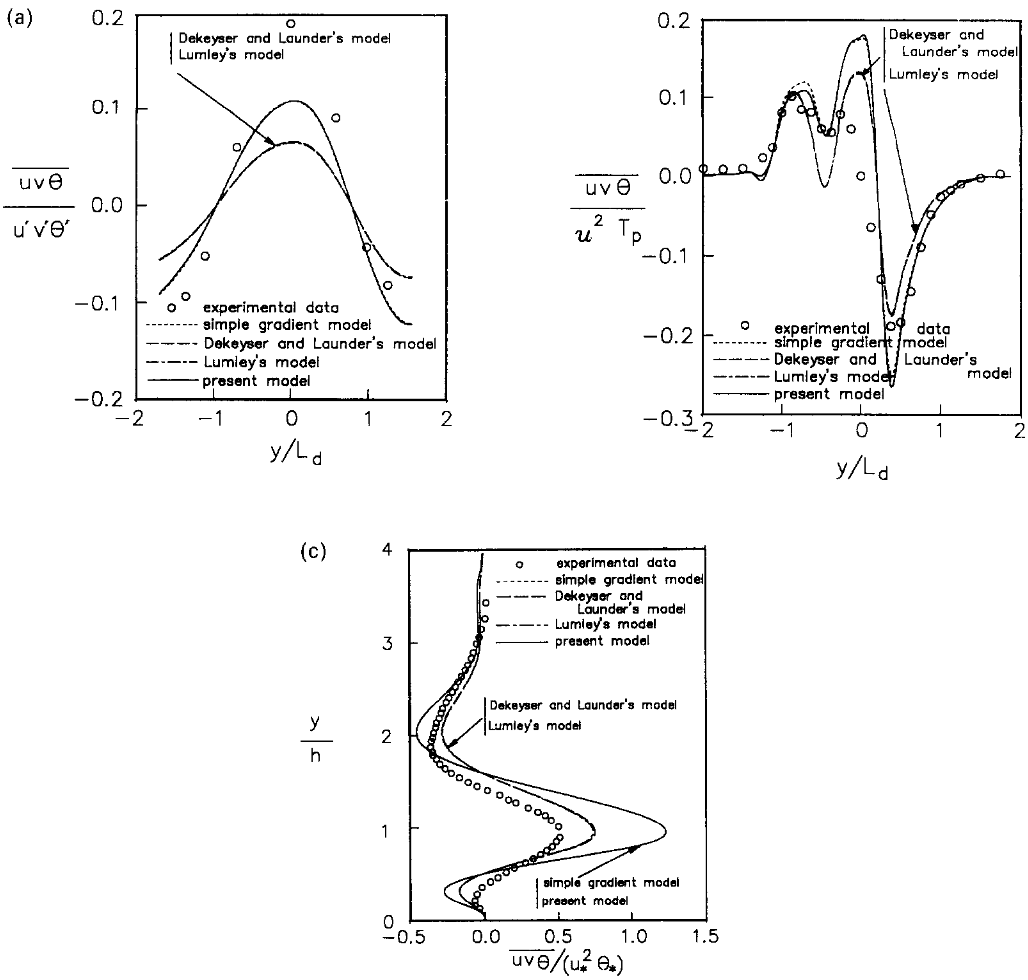

FIG. 3. (a) Model predictions and experimental (Karnik and Tavoularis' [21]) data for the streamwise transport of the vertical heat flux, $\overline{uv\theta}$. (b) Model predictions and experimental (Chung and Kyong's [3]) data for the streamwise transport of the vertical heat flux, $\overline{uv\theta}$. (c) Model predictions and experimental (Raupach and Legg's [2]) data for the streamwise transport of the vertical heat flux, $\overline{uv\theta}$.

able shear as in Figs. 3(b) and (c), the addition of the shear-gradient contribution also improves the computation of $\overline{uv\theta}$.

4. CONCLUSIONS

In order to investigate the comparative performances of various computational turbulence models for third-order diffusive terms, $\overline{u_iu_j\theta}$ and $\overline{u_i\theta^2}$, which appear in the second-order equations for the kinematic heat flux, $\overline{u_j\theta}$, and the temperature variance, $\overline{\theta^2}$, respectively, the conventional simple gradient model, two eddy-damped quasi-normal approximation models and Weinstock's theoretical model based on a formal integration of the Navier–Stokes equation are summarized and they are applied to a number of recent measurements in non-isothermal turbulent flow fields. All second-order quantities and the mean temperature data necessary to evaluate the models are taken from the directly measured values. Available third-order data from the selected measurements are $\overline{v\theta^2}$, $\overline{v^2\theta}$ and $\overline{uv\theta}$. The major concern of the present study was to see if increasing the complexity of the model (or using less approximation in the model derivation) led to better prediction or not.

The comparisons between data and the various predictions for the scalar variance diffusion term, $\overline{u_i\theta^2}$, reveal a surprising result that the simple gradient-type model has the best overall prediction performance among the models tested here. In addition, a method to modify the simple gradient-type model is successfully proposed to correctly include the buoyancy effect in the model by introducing a time scale ratio between the turbulence and the buoyancy, and by replacing the conventional Reynolds stress in the simple gradient model with an effective buoyant Reynolds stress,

$$\overline{u_iu_j}_{\text{eff}} = \overline{u_iu_j} - b\beta(g_i\overline{u_j\theta} + g_j\overline{u_i\theta})\tau_\theta.$$

Such a modification has been shown to shift the predicted profiles in desirable directions for both thermally stable and unstable layers.

However, for the scalar flux diffusion terms, $\overline{u_i u_j \theta}$, it is found that the contribution of the shear-gradient term must be included in the model in predicting turbulent flows with an appreciable shear. Here, again, the same buoyancy modification to the eddy coefficient of the model (3) must improve the predictions of the non-negligible buoyant flow field.

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ETUDE COMPARATIVE DE PLUSIEURS MODELES DE CALCUL POUR LES MOMENTS TRIPLES DE VITESSE ET DE SCALAIRE DANS UNE FERMETURE DE SECOND ORDRE

Résumé—Une évaluation comparative de plusieurs modèles de turbulence pour les termes de troisième ordre du transport diffusif, $\overline{u_i u_j \theta}$ et $\overline{u_i \theta^2}$, dans les équations de fermeture au second ordre, a été conduite en appliquant les modèles à plusieurs écoulements turbulents non isothermes. Les quantités de second ordre apparaissant dans les modèles sont adoptés à partir des valeurs mesurées directement. Les modèles testés sont le modèle conventionnel simple à gradient, le modèle avec approximation quasi-normale d'amortissement turbulent et le modèle théorique de Weinstock qui est dérivé d'une intégration formelle de l'équation de Navier–Stokes [J. Weinstock, *J. Fluid Mech.* **202,** 319–338 (1989)]. Il est étonnant de constater que le modèle simple de gradient conduit à des résultats équivalents ou même meilleur que d'autres plus compliqués pour la diffusion de la variance d'un scalaire $\overline{u_i \theta^2}$. Néanmoins on trouve que le modèle de calcul de la diffusion de flux de scalaire $\overline{u_i u_j \theta}$ doit inclure la contribution du gradient cisaillant en plus du modèle simple de gradient. Une méthode de correction de flottement est proposée pour tenir compte de l'effet de flottement dans les modèles de type gradient.

VERGLEICHENDE UNTERSUCHUNG VERSCHIEDENER RECHENMODELLE FÜR DIE DREIFACHMOMENTE DER GESCHWINDIGKEIT UND SKALARER GRÖSSEN BEI DER SCHLIESSBEDINGUNG ZWEITER ORDNUNG

Zusammenfassung—Verschiedene rechnerische Turbulenzmodelle für die diffusiven Transportterme dritter Ordnung ($\overline{u_u u_h \theta}$ und $\overline{u_i \theta^2}$) in den Schließgleichungen zweiter Ordnung wurden vergleichend betrachtet, indem die Modelle auf verschiedene nichtisotherme turbulente Strömungen angewandt wurden. Die Größen zweiter Ordnung in diesen Modellen wurden von direkt gemessenen Werten übernommen. Folgende Modelle wurden in der vorliegenden Arbeit untersucht: das gewöhnliche einfache Gradientenmodell; das quasi-normale Näherungsmodell mit Wirbeldämpfung und das theoretische Modell nach Weinstock, welches durch formale Integration der Navier-Stokes-Gleichung hergeleitet wird (J. Weinstock, *J. Fluid Mech.* **202,** 319–338 (1989)). Es ist recht erstaunlich, daß sich das einfache Gradientenmodell den komplizierteren Modellen ebenbürtig erweist oder sogar besser ist, im Hinblick auf die Diffusion von $\overline{u_i \theta^2}$. Es zeigt sich jedoch, daß das Rechenmodell für den skalaren Fluß $\overline{u_i u_j \theta}$ zusätzlich zum einfachen Gradientenmodell den Beitrag des Schergradienten enthalten muß. Es wird zusätzlich ein Verfahren zur Auftriebskorrektur vorgestellt, das es erlaubt den Auftriebseinfluß in den Modellen vom Gradiententyp zu berücksichtigen.

СРАВНИТЕЛЬНОЕ ИССЛЕДОВАНИЕ РАЗЛИЧНЫХ РАСЧЕТНЫХ МОДЕЛЕА ДЛЯ МОМЕНТОВ ТРЕТЬЕГО ПОРЯДКА ОТНОСИТЕЛЬНО СКОРОСТИ И СКАЛЯРА В ЗАМЫКАНИЯХ ВТОРОГО ПОРЯДКА

Аннотация—Проводится сравнительная оценка нескольких расчетных моделей турбулентности для членов диффузионного переноса третьего порядка $\overline{u_i u_j \theta}$ и $\overline{u_i \theta^2}$ в уравнениях замыкания второго порядка посредством их применения к различным неизотермическим турбулентным течениям. Входящие в модели величины второго порядка взяты из прямых измерений. Апробируются следующие модели: общепринятая простая градиентная модель, модель квазинормальных приближений вихревого затухания, а также теоретическая модель Вейнстока, полученная путем формального интегрирования уравнения Навье-Стокса (J. Weinstock *J. Fluid Mech.* **202**, 319–338 (1989)). Неожиданным результатом явилось то, что простая градиентная модель так же или даже более эффективна при расчете диффузии скалярной дисперсии $\overline{u_i \theta^2}$, чем другие более сложные. Однако найдено, что расчетная модель для диффузии скалярного потока $\overline{u_i u_j \theta}$ должна учитывать вклад сдвиговых градиентов в дополнение к градиентной модели. Кроме того, для учета эффекта подъемной силы в моделях градиентного типа предложен соответствующий метод поправки.